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Research Article

Fuzzy Model With Non-Linear Demand Pattern Using Preservation Technology With Non-Linear Holding Cost To Control Deterioration Rate

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Abstract: The demand for a product is one of the important components of Inventory management. Most of the time, it is not constant and can occasionally change based on several important variables. Demand is seen to be influenced by both stock levels and selling prices for any given product. The terms "selling" and "stock-based" refer to this kind of demand. Inventory constraints are not usually set, sometimes because of consumer uncertainty. Regarding inventory control, a few factors are crucial, including demand, holding costs, deterioration costs, lost sale costs, shortage costs, purchase costs, etc. This article develops a fuzzy economic order quantity (EOQ) inventory model that allows for shortage with partial backlog, given that these parameters are unknown in real-world scenarios. Technological preservation is used to reduce the pace of deterioration. Fuzzy triangular numbers are used as parameters to find the fuzzy total average cost function, and the Graded Mean Integration Representation (GMIR), Signed Distance, and Centroid methods are used to defuzzify the model. To prove the authenticity of the new model through a numerical example also discussed the sensitivity analysis which shows the impact of different parameters.

Keywords: Selling price- stock dependent demand, preservation technology, deterioration, triangular fuzzy number, GMIR

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Introduction:
The vast majority of businesses depend on customer demand no firm will ever bother to care about manufacturing without a demand. Additionally, stock and sales prices are strongly correlated with market demand. As a result, stock and sales prices are the two factors that directly impact revenue. In other words, higher sale rates and a lack of inventory reduce demand, whereas lower ones have the opposite impact. As a result, applying an appropriate apprisingandstockstrategymakesiteasyforbusinessesto managetheirinventorysystems.This model considers a non-linear function of the sales price and stock. In this paper, we worked on decaying items like vegetables, etc. The inventory model for decaying products has received a lot of attention recently.

It is noticed that vegetable items are sometimes available in the market at very low prices not at all due to lack of cold storage. At harvesting time in some places, the vegetables like tomato, onion, and cauliflower, the price is low and the farmers sometimes left the product in the field itself for destruction. However, the prices of these vegetables are very high in other unseasonal months of the year. In accounting for these difficulties, we developed a model of deteriorating products using preservation technology cost. Here we considered that the above items can be calculated from local farmers, stored using preservation technology cost, and sold at their peak value. Finally, the basic price of the product is to be given to the farmers from which they come out. In this model, we use fuzzy technology since the parameters like demand, holding cost, deterioration cost, lost sale cost, shortage cost, and purchase cost probably will have some little fluctuations or are vague in nature. So, in these practical situations, we treat these parameters as triangular fuzzy variables which will be more realistic. Hence fuzzy set theory is necessary for the formulation of such inventory models.

Zadeh was the first person who introduced fuzziness in (1965) after that in 1970 Zadeh and Bellman used fuzzy set theory to develop an inventory model. It gives accurate results Compared to probability theory, decision theory, Control theory, etc. Indrajitsingha etal. (2018) developed an EOQ model with a demand rate in one Case as the stock base and in another Case as Constant, holding cost, demand, and deterioration rate taken as triangular fuzzy numbers and defuzzify the total cast function by using GMIR, SD and centroid method. Indrajitsinghal, S.K. et al. (2021) proposed a fuzzy model for deteriorating items, and due to uncertainty in parameters using triangular fuzzy numbers. To control deterioration the preservation technology has been in the fuzzy model. Kuppulakshmia V. et al (2023) evolved a fuzzy economic manufacturing model for the imperfect production process. The maintenance cost of the products has increased during the pandemic, and products accumulated without sale. Allow the special sale of products with discount and without discount prices.

Deterioration plays an important role in developing an inventory model for seasonal products like fruits and vegetables etc. To control the deterioration technique preservation for the seasonal products due to which preservation costs arise and get more profit in comparison to those models developed without preservation. During the last decade of the present century, the use of preservation technology cost while developing an inventing model, considered at various times, had been by different researchers Hsu, P.H. et al. (2010), Dye, cay, et al (2012, 2013), Mishra, U. (2018), Saha, S. et al. (2017), Das. S.C. etal. (2020), Sahu, A.K. et al. (2020), The authors A.K. Sahu et al. (2024) have a fuzzy model as well as a crisp model with production and selling price-dependent demand. SumeetGill and Kumari, M. (2024) developed a model for Seasonal inventory products like vegetables with price and stock-dependent demand and non-linear holding cost. To Control the deterioration of products the retailer used preservation technology they suggested adopting fuzziness due to uncertainty like products to meet the reality of inventory and could be an extension of their model. Few studies have also used preservation technology with stock, and selling price-based demand also using constant and linear holding cost is fuzzy inventory models with backorder difficulties in the relevant literature that is currently available. In this present paper, we develop a fuzzy model with non-linear holding cost and control deterioration by using preservation technology.

Literature Review

Demand is the most important factor for the majority of businesses. Most businesses depend on the customer's demand. A high demand factor encourages an organization to produce more. It is observed that market demand directly affects the selling price and stock of the item. In short, the demand rate is higher by lowered sales price and enough availability of stock. Using a proper strategy improves the convenience of businesses in handling inventory systems, and in this mode considers a non-linear function of sales price and stock: Moreover, the holding cost is considered as a non-linear quadratic function of time. Conversely, shortages are often considered in many realistic inventory control schemes, where the demand can be backlogged until the order is re-filled in the scheme, or it may be skipped depending on the customer's preference and its products. A Standard Economic Order Quantity model was developed by F.W. Harris (1915) this model was solved by using two methods i.e. Tabulation Method and Algebraic Method. P.H. Hsu et al (2010) examined the preservation technology Investment for decay items with a constant demand rate and deterioration rate. Singh S.R., et al. (2016) developed an economic ordered quantity model for decay products influenced by stock level i.e. demand is stock-based. Similarly, Mishra V. et al (2019), Raula, P. (2021), and Sindhija S et al. (2023) developed inventory models with stock-dependent demand.

Model Notation and Assumptions

Notations

Throughout the paper, we use the following notations:

$D(S,q(t))$: Demand rate

α: Initial demand rate.

β: Positive demand coefficient.

$n(\tau)$: Deterioration rate with investment in preservation technology.

n_o: Deterioration rate without investment in preservation technology.

η: Sensitive parameter of investment to the deterioration rate.

$q(t)$: On hand stock at time t.

q: Initial Inventory level.

S: Selling price.

C_{hc}: Holding cost.

C_{sc}: Shortage cost.

C_{lc}: Lost sale cost

C_{dc}: Deterioration cost.

C_{oc}: Ordering cost.

C_{pc}: Purchasing cost.

T: The duration of the replenishment cycle.

$TAC(\tau,t_1)$: Total average cost for inventory management $0\le t_1\le T$.

$\tilde{\alpha}$: Fuzzy initial demand rate.

$\tilde{\beta}$: Fuzzy positive demand rate.

$\tilde{S}$: Fuzzy selling price.

$\tilde{C}_{hc}$: Fuzzy holding cost.

$\tilde{C}_{sc}$: Fuzzy shortage cost.

$\tilde{C}_{lc}$: Fuzzy lost sell cost.

$\tilde{C}_{dc}$: Fuzzy deterioration cost.

$\widetilde{C_{pc}}$: Fuzzy purchasing cost.

$\widetilde{TAC}(\tau, t_1)$: Fuzzy total average cost.

$\widetilde{TAC_g}(\tau, t_1)$: Defuzzified value of $\widetilde{TAC}(\tau, t_1)$ with GMIR method ($0\le t_1\le T$).

$\widetilde{TAC_S}(\tau, t_1)$: Defuzzified value of $\widetilde{TAC}(\tau, t_1)$ with SDM method ($0\le t_1\le T$).

$\widetilde{TAC_C}(\tau, t_1)$: Defuzzified value of $\widetilde{TAC}(\tau, t_1)$ with CM method ($0\le t_1\le T$).

Assumptions:

The inventory model is based on the following assumptions:

i. The demand for the product depends on price and stock, for the price-dependent function is considered as linear.

ii. The inventory system involves only a single product.

iii. The lead time is assumed to be negligible.

iv. Shortages are allowed with partial backlogging and the backlogging rate is $e^{-\lambda(T-t)}$; λ is the backlogging variable and positive constant.

v. It is assumed that $n(\tau) = n_o e^{-\eta\tau}$ and $n(\tau)$ is the rate of deterioration without investment in preservation technology and η is a sensitive parameter of investment to the deterioration rate.

vi. The rate of replenishment is finite.

vii. The holding cost is non-linear and modeled with a quadratic function that depends on time i.e. $h+h_1t+h_2t^2$.

Mathematical Model:

In this section, we formulate a mathematical model for the inventory system. Products in the system are deteriorating at a constant rate and Preservation technology helps retailers to reduce the deterioration rate. Moreover, inventory level decreases due to product demand. Hence, the inventory level at any time t is governed by the differential equation given below. Thus, the inventory system can be described by the following differential equations:

$$\frac{dq(t)}{dt}+n(\tau)q(t)=-D(S,q(t)) \;;0\le t\le t_1 \qquad (1)$$

$$\frac{dq(t)}{dt}=-D(S,q(t))e^{-\lambda(T-t)} \;;t_1\le t\le T \qquad (2)$$

With the boundary conditions

$$q(0)=q,\ q(t_1)=0 \qquad (3)$$

The solutions of these equations are given by

$$q(t)=\frac{d(s)}{(n(\tau)+c)}\left[e^{(n(\tau)+c)(t_1-t)}-1\right];0\le t\le t_1 \qquad (4)$$

$$q(t)=\frac{d(s)}{\lambda}\left[e^{-\lambda(T-t_1)}-e^{-\lambda(T-t)}\right];t_1\le t\le T \qquad (5)$$

$$q=\frac{d(s)}{(n(\tau)+c)}\left[e^{(n(\tau)+c)t_1}-1\right] \qquad (6)$$

By using the equation (4), (5) and (6) the value of parameters has been calculated

Now, the total cost is calculated by using the following basic costs:

1) **Ordering Cost**

$OC = C_{oc}$ (7)

2) **Holding cost**

$$HC=\frac{d(s)}{\left(n(\tau)+c\right)^{2}}\left[\begin{array}{l} h\left\{e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right\}+\frac{h_1}{\left(n(\tau)+c\right)}\left\{e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right\} \\ +\frac{h_2}{\left(n(\tau)+c\right)^{2}}\left\{2e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)^{2}t_1^{2}-2\left(n(\tau)+c\right)t_1-2\right\} \\ -\left(n(\tau)+c\right)\left\{\frac{t_1^2h_1}{2}-\frac{t_1^3h_2}{3}\right\} \end{array}\right] \tag{8}$$

3) **Purchasing cost**

$$PC = C_{pc}q$$

$$= C_{pc}\frac{d(s)}{\left(n(\tau)+c\right)}\left[e^{(n(\tau)+c)t_1}-1\right] \tag{9}$$

4) Shortage cost

$$SC=-C_{sc}\frac{d(s)}{\lambda^{2}}\left[\lambda\left(T-t_1\right)e^{-\lambda(T-t_1)}-1+e^{-\lambda(T-t_1)}\right] \tag{10}$$

5)Lost Sale cost

$$LSC=-C_{lc}\frac{d(s)}{\lambda}\left[\lambda\left(T-t_1\right)-1+e^{-\lambda(T-t_1)}\right] \tag{11}$$

6) Deterioration cost

$$DC=C_{dc}\frac{d(s)n(\tau)}{\left(n(\tau)+c\right)^{2}}\left[e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right] \tag{12}$$

6) Preservation technology cost

$$PTC=\tau T \tag{13}$$

Average total cost $TAC(\tau,t_1)$ for the period T

$$TAC(\tau,t_1)=\frac{1}{T}[HC+DC+OC+PC+SC+LSC+PTC] \tag{14}$$

$$=\left\{\begin{array}{l} C_{oc}+\frac{d(s)}{\left(n(\tau)+c\right)}\left[\begin{array}{l} h\left\{e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right\}+\frac{h_1}{\left(n(\tau)+c\right)^{2}}\left\{e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right\} \\ +\frac{h_2}{\left(n(\tau)+c\right)^{3}}\left\{2e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)^{2}t_1^{2}-2\left(n(\tau)+c\right)t_1-2\right\}-\left\{\frac{t_1^2h_1}{2}-\frac{t_1^3h_2}{3}\right\} \end{array}\right] \\ +C_{dc}\frac{d(s)n(\tau)}{\left(n(\tau)+c\right)^{2}}\left[e^{(n(\tau)+c)t_1}-\left(n(\tau)+c\right)t_1-1\right]+C_{pc}\frac{d(s)}{\left(n(\tau)+c\right)}\left[e^{(n(\tau)+c)t_1}-1\right] \\ -C_{sc}\frac{d(s)}{\lambda^{2}}\left[\lambda\left(T-t_1\right)e^{-\lambda(T-t_1)}-1+e^{-\lambda(T-t_1)}\right]-C_{lc}\frac{d(s)}{\lambda}\left[\lambda\left(T-t_1\right)-1+e^{-\lambda(T-t_1)}\right]+\tau T \end{array}\right\} \tag{15}$$

For a small x value, the Taylor series says that the exponential function has an approximation of $e^{x}\approx 1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}$ using this result in the above equation.

$$TAC(\tau,t_1)=\frac{1}{T}\left[\begin{array}{l}(C_{oc}+\tau T)+\\ d(s)\left\{\begin{array}{l}\left(\left(h+\frac{h_1}{(n(\tau)+c)}\right)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+h_2\frac{t_1^3}{6}-\frac{1}{(n(\tau)+c)}\left(h_1\frac{t_1^2}{2}+h_2\frac{t_1^3}{6}\right)\right)\\ +C_{dc}n(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+C_{pc}(n(\tau)+c)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ -C_{sc}(T-t_1)^2\left(-\frac{1}{2}+\frac{\lambda}{3}(T-t_1)-\frac{\lambda^2}{6}(T-t_1)^2\right)-C_{lc}\frac{\lambda}{2}(T-t_1)^2\left(1-\frac{\lambda}{3}(T-t_1)\right)\end{array}\right\}\end{array}\right] \quad (16)$$

Where $d(s)=\alpha-\beta s,\ n(\tau)=n_o e^{-\eta\tau}$

Following the necessary conditions to minimize the average total cost function TAC(τ,T_1) per unit of time, the value of τ and t_1 that minimizes average cost can be obtained by solving the equation.

$$\frac{\partial TAC(\tau,t_1)}{\partial\tau}=0 \text{ and } \frac{\partial TAC(\tau,t_1)}{\partial t_1}=0 \quad (16)$$

Satisfying the conditions

$$\frac{\partial^2 TAC(\tau,t_1)}{\partial\tau^2}>0\ ,\ \frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2}>0,\text{ and}$$

$$\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial\tau^2}\right)\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial t_1^2}\right)-\left(\frac{\partial^2 TAC(\tau,t_1)}{\partial\tau\partial t_1}\right)^2>0 \quad (17)$$

Equations (16) are equivalent to

$$\frac{1}{T}\left[d(s)\left\{\begin{array}{l}-\frac{n'(\tau)h_1}{(n(\tau)+c)^2}\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+\left(h+\frac{h_1}{(n(\tau)+c)}\right)\left(n'(\tau)\frac{t_1^3}{6}\right)\\ +\frac{n'(\tau)}{(n(\tau)+c)^2}\left(h_1\frac{t_1^2}{2}+h_2\frac{t_1^3}{6}\right)+C_{pc}n'(\tau)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ +C_{dc}\left(n'(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+n(\tau)n'(\tau)\frac{t_1^3}{6}\right)\end{array}\right\}\right]=0$$

$$\frac{1}{T}\left[d(s)\left\{\begin{array}{l}\left(\left(h+\frac{h_1}{(n(\tau)+c)}\right)\left(t_1+(n(\tau)+c)\frac{t_1^2}{2}\right)+h_2\frac{t_1^2}{2}-\frac{1}{(n(\tau)+c)}\left(h_1t_1+h_2\frac{t_1^2}{2}\right)\right)\\ +C_{dc}n(\tau)\left(t_1+(n(\tau)+c)\frac{t_1^2}{2}\right)+C_{pc}(n(\tau)+c)\left(3t_1+\frac{t_1^2}{2}\right)\\ -C_{sc}(T-t_1)\left(-1+\frac{\lambda}{3}(T-t_1)+\frac{2\lambda^2}{3}(T-t_1)^2\right)+C_{lc}\lambda(T-t_1)\left(1-\frac{\lambda}{2}(T-t_1)\right)\end{array}\right\}\right]=0$$

Fuzzy Model

Due to uncertainty, it is not easy to define all the system of parameters exactly. Subsequently, we assume them as fuzzy parameters, namely $\tilde{\alpha}$,$\tilde{\beta}$,$\tilde{S}$,$\tilde{C}_{hc}$,$\tilde{C}_{sc}$,$\tilde{C}_{lc}$,$\tilde{C}_{dc}$, and $\tilde{C}_{pc}$. These parameters may change within some limits. $\tilde{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$, $\tilde{\beta}=(\beta_1,\beta_2,\beta_3)$, $S=(S_1,S_2,S_3)$, $\tilde{h}=(a_1,a_2,a_3)$, $\tilde{h}_1=(b_1,b_2,b_3)$, $\tilde{h}_2=(c_1,c_2,c_3)$, $\tilde{C}_{sc}=(s_1,s_2,s_3)$, $\tilde{C}_{lc}=(l_1,l_2,l_3)$, $\tilde{C}_{dc}=(d_1,d_2,d_3)$, and $\tilde{C}_{pc}=(p_1,p_2,p_3)$ be considered as triangular fuzzy numbers.

$$\widetilde{TAC}(\tau,t_1)=\frac{1}{T}\left[\begin{array}{l}(C_{oc}+\tau T)+\\ (\tilde{\alpha}+\tilde{\beta}\tilde{S})\left\{\begin{array}{l}\left(\left(\tilde{h}+\frac{\tilde{h}_1}{(n(\tau)+c)}\right)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+\tilde{h}_2\frac{t_1^3}{6}-\frac{1}{(n(\tau)+c)}\left(\tilde{h}_1\frac{t_1^2}{2}+\tilde{h}_2\frac{t_1^3}{6}\right)\right)\\ +\tilde{C}_{dc}n(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+\tilde{C}_{pc}(n(\tau)+c)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ -\tilde{C}_{sc}(T-t_1)^2\left(-\frac{1}{2}+\frac{\lambda}{3}(T-t_1)-\frac{\lambda^2}{6}(T-t_1)^2\right)-\tilde{C}_{lc}\frac{\lambda}{2}(T-t_1)^2\left(1-\frac{\lambda}{3}(T-t_1)\right)\end{array}\right\}\end{array}\right] \tag{18}$$

Defuzzify the fuzzy total average cost. $\widetilde{TAC}(\tau,t_1)$ by GMIR method:

$$\widetilde{TAC}_g(\tau,t_1)=\frac{1}{6}[\widetilde{TAC}_{g_1}(\tau,t_1)+4\widetilde{TAC}_{g_2}(\tau,t_1)+\widetilde{TAC}_{g_3}(\tau,t_1)] \tag{19}$$

$$\widetilde{TAC}_{g_i}(\tau,t_1)=\frac{1}{T}\left[\begin{array}{l}(C_{oc}+\tau T)+\\ (\alpha_i+\beta_i S_i)\left\{\begin{array}{l}\left(\left(a_i+\frac{b_i}{(n(\tau)+c)}\right)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+c_i\frac{t_1^3}{6}-\frac{1}{(n(\tau)+c)}\left(b_i\frac{t_1^2}{2}+c_i\frac{t_1^3}{6}\right)\right)\\ +d_i n(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+p_i(n(\tau)+c)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ -s_i(T-t_1)^2\left(-\frac{1}{2}+\frac{\lambda}{3}(T-t_1)-\frac{\lambda^2}{6}(T-t_1)^2\right)-l_i\frac{\lambda}{2}(T-t_1)^2\left(1-\frac{\lambda}{3}(T-t_1)\right)\end{array}\right\}\end{array}\right]$$

for i=1,2,3 (20)

To minimize the total average cost function $\widetilde{TAC}_g(\tau,t_1)$ per unit time, the optimal value of τ and t_1 can be obtained by solving the equations

$$\frac{\partial\widetilde{TAC}_g(\tau,t_1)}{\partial\tau}=0 \quad\text{and}\quad \frac{\partial\widetilde{TAC}_g(\tau,t_1)}{\partial t_1}=0 \tag{21}$$

Satisfying the equations

$$\frac{\partial^2\widetilde{TAC}_g(\tau,t_1)}{\partial\tau^2}>0\ ,\ \frac{\partial^2\widetilde{TAC}_g(\tau,t_1)}{\partial t_1^2}>0,\ \text{and}$$
$$\left(\frac{\partial^2\widetilde{TAC}_g(\tau,t_1)}{\partial\tau^2}\right)\left(\frac{\partial^2\widetilde{TAC}_g(\tau,t_1)}{\partial t_1^2}\right)-\left(\frac{\partial^2\widetilde{TAC}_g(\tau,t_1)}{\partial\tau\partial t_1}\right)^2>0 \tag{22}$$

Defuzzify the fuzzy total average cost $\widetilde{TAC}(\tau,t_1)$ by SDM method:

$$\widetilde{TAC}_S(\tau,t_1)=\frac{1}{4}[\widetilde{TAC}_{S_1}(\tau,t_1)+2\widetilde{TAC}_{S_2}(\tau,t_1)+\widetilde{TAC}_{S_3}(\tau,t_1)] \tag{23}$$

$$\widetilde{TAC_{S_i}}(\tau,t_1)=\frac{1}{T}\left[\begin{array}{l}(C_{oc}+\tau T)+\\ (\alpha_i+\beta_i S_i)\left\{\begin{array}{l}\left(\left(a_i+\frac{b_i}{(n(\tau)+c)}\right)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+c_i\frac{t_1^3}{6}-\frac{1}{(n(\tau)+c)}\left(b_i\frac{t_1^2}{2}+c_i\frac{t_1^3}{6}\right)\right)\\ +d_i n(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+p_i(n(\tau)+c)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ -s_i(T-t_1)^2\left(-\frac{1}{2}+\frac{\lambda}{3}(T-t_1)-\frac{\lambda^2}{6}(T-t_1)^2\right)-l_i\frac{\lambda}{2}(T-t_1)^2\left(1-\frac{\lambda}{3}(T-t_1)\right)\end{array}\right\}\end{array}\right]$$

For i=1,2,3 (24)

To minimize the total average cost function $\widetilde{TAC_S}(\tau,t_1)$ per unit time, the optimal value of τ and t_1 can be obtained by solving the equations

$$\frac{\partial\widetilde{TAC_S}(\tau,t_1)}{\partial\tau}=0 \text{ and } \frac{\partial\widetilde{TAC_S}(\tau,t_1)}{\partial t_1}=0 \quad (25)$$

Satisfying the equations

$$\frac{\partial^2\widetilde{TAC_S}(\tau,t_1)}{\partial\tau^2}>0\ ,\ \frac{\partial^2\widetilde{TAC_S}(\tau,t_1)}{\partial t_1^2}>0, \text{ and}$$
$$\left(\frac{\partial^2\widetilde{TAC_S}(\tau,t_1)}{\partial\tau^2}\right)\left(\frac{\partial^2\widetilde{TAC_S}(\tau,t_1)}{\partial t_1^2}\right)-\left(\frac{\partial^2\widetilde{TAC_S}(\tau,t_1)}{\partial\tau\partial t_1}\right)^2>0 \quad (26)$$

Defuzzify the fuzzy total average cost $\widetilde{TAC}(\tau,t_1)$ by CM method:

$$\widetilde{TAC_C}(\tau,t_1)=\frac{1}{3}[\widetilde{TAC_{C_1}}(\tau,t_1)+\widetilde{TAC_{C_2}}(\tau,t_1)+\widetilde{TAC_{C_3}}(\tau,t_1)] \quad (27)$$

$$\widetilde{TAC_{C_i}}(\tau,t_1)=\frac{1}{T}\left[\begin{array}{l}(C_{oc}+\tau T)+\\ (\alpha_i+\beta_i S_i)\left\{\begin{array}{l}\left(\left(a_i+\frac{b_i}{(n(\tau)+c)}\right)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+c_i\frac{t_1^3}{6}-\frac{1}{(n(\tau)+c)}\left(b_i\frac{t_1^2}{2}+c_i\frac{t_1^3}{6}\right)\right)\\ +d_i n(\tau)\left(\frac{t_1^2}{2}+(n(\tau)+c)\frac{t_1^3}{6}\right)+p_i(n(\tau)+c)\left(\frac{3}{2}t_1^2+\frac{t_1^3}{6}\right)\\ -s_i(T-t_1)^2\left(-\frac{1}{2}+\frac{\lambda}{3}(T-t_1)-\frac{\lambda^2}{6}(T-t_1)^2\right)-l_i\frac{\lambda}{2}(T-t_1)^2\left(1-\frac{\lambda}{3}(T-t_1)\right)\end{array}\right\}\end{array}\right]$$

For i=1,2,3 (28)

To minimize the total average cost function $\widetilde{TAC_C}(\tau,t_1)$ per unit time, the optimal value of τ and t_1 can be obtained by solving the equations.

$$\frac{\partial\widetilde{TAC_C}(\tau,t_1)}{\partial\tau}=0 \text{ and } \frac{\partial\widetilde{TAC_C}(\tau,t_1)}{\partial t_1}=0 \quad (29)$$

Satisfying the equations

$$\frac{\partial^2\widetilde{TAC_C}(\tau,t_1)}{\partial\tau^2}>0\ ,\ \frac{\partial^2\widetilde{TAC_C}(\tau,t_1)}{\partial t_1^2}>0, \text{ and}$$
$$\left(\frac{\partial^2\widetilde{TAC_C}(\tau,t_1)}{\partial\tau^2}\right)\left(\frac{\partial^2\widetilde{TAC_C}(\tau,t_1)}{\partial t_1^2}\right)-\left(\frac{\partial^2\widetilde{TAC_C}(\tau,t_1)}{\partial\tau\partial t_1}\right)^2>0 \quad (30)$$

Numerical Example

To illustrate the result of the proposed model, we consider the numerical example of the inventory system with the following parametric values. Consider a real scenario taking some data from vegetable farmhouse and using some secondary data to prove the authenticity of the model.

Crisp Model

Let us suppose

$\alpha = 1000\text{units}; \beta = 10;\ S = 30\text{Rs./Unit}; T = 13\text{weeks}; \eta = 0.4; \lambda = 0.04; n_o = 4;\ h = 1.75\text{Rs./Unit/week}^2;$

$h_1 = 0.15\text{Rs./Unit/week}^2; h_2 = 0.25\text{Rs./Unit/week}^3;\ C_{pc} = 10\text{Rs./Unit};\ C_{oc} = 250;\ C_{sc} = 10\text{Rs./Unit};$

$C_{dc} = 4\text{Rs./Unit};\ C_{lc} = 9\text{Rs./Unit};\ c = 0.5;$

By using Mathematica, we calculate the optimal solution for the inventory system and we get the optimal solution.

$\tau^* = \text{Rs.} 2.37803,\ t_1^* = 0.712086\text{weeks}, q^* = 942.988$, and $TAC^* = \text{Rs.}\ 30785.9$

To show the convexity of the total cost function $TAC(\tau, t_1)$, we plot a 3D graph shown in Figure 1

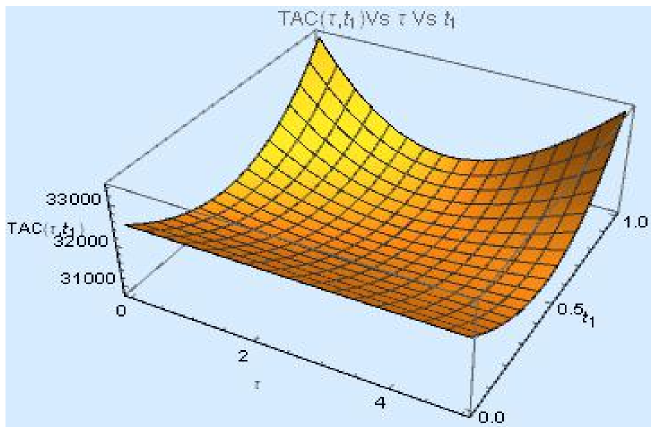

Fig.1Graphical representation of convex optimization of total cost function with Preservation Cost τ and shortage and shortage period t_1

Fuzzy Model

Let us consider

$\tilde{\alpha} = (800, 900, 1000), \tilde{\beta} = (8, 10, 12), \tilde{S} = (24, 30, 36), \tilde{h} = (1.5, 1.75, 2), \tilde{h}_1 = (0.1, 0.15, 0.2), \tilde{h}_2 = (0.2, 0.25, 0.3), \tilde{C}_{pc} = (9, 10, 11),$

$\tilde{C}_{sc} = (7, 10, 13), \tilde{C}_{dc} = (3, 4, 5), \tilde{C}_{lc} = (6, 9, 12), C_{oc} = 250; T = 13\text{weeks}; \eta = 0.4; \lambda = 0.04; n_o = 4; c = 0.5$

We get the fuzzy total average cost, determined by the GMIR method, is

$\tilde{\alpha} = (800, 900, 1000), \tilde{\beta} = (8, 10, 12), \tilde{S} = (24, 30, 36), \tilde{h} = (1.5, 1.75, 2), \tilde{h}_1 = (0.1, 0.15, 0.2), \tilde{h}_2 = (0.2, 0.25, 0.3), \tilde{C}_{pc} = (9, 10, 11),$

$\tilde{C}_{sc} = (7, 10, 13), \tilde{C}_{dc} = (3, 4, 5), \tilde{C}_{lc} = (6, 9, 12), C_{oc} = 250; T = 13\text{weeks}; \eta = 0.4; \lambda = 0.04; n_o = 4; c = 0.5$

$\widetilde{TAC}_g(\tau, t_1) = \text{Rs.}26129.9$, with $\tau = \text{Rs.}2.37833, t_1 = 0.711088\text{weeks}$.

by the SDM method is $\widetilde{TAC}_S(\tau, t_1) = \text{Rs.}25999.4$, with $\tau = \text{Rs.}2.37834, t_1 = 0.710578\text{weeks}$.

by the CM method is $\widetilde{TAC}_C(\tau, t_1) = \text{Rs.}25868.8$, with $\tau = \text{Rs.}2.37915, t_1 = 0.710064\text{weeks}$.

Sensitivity analysis

The behavior of the system factors that affect the average total cost function is crucial information for retailers using inventory systems. The retailer has to be aware of the point at which a drop or increase in the relevant parameters results in the lowest possible expense. Thus, we investigate sensitivity analysis with the change of several parameters to show the applicability of the model and identify some important management implications in vegetable farmhouses.

From Figure 2 to Figure 7, we studied the system parameters with various values in a fuzzy sense, while leaving certain parameters at their original levels.

Table 1. Sensitivity analysis on initial demand

	GIMR			SDM			CM		
α	τ	t_1	$\widetilde{TAC}_g(\tau,t_1)$	τ	t_1	$\widetilde{TAC}_S(\tau,t_1)$	τ	t_1	$\widetilde{TAC}_C(\tau,t_1)$
(850,900, 950)	2.3804	0.70855	25916.7	2.38188	0.70675	25679.5	2.38337	0.70492	25442.3
(900,950,	2.3805	0.70883	28114.1	2.38182	0.70716	27876.9	2.38319	0.70548	27639.7

1000)									
(950,1000,1050)	2.3805	0.70906	30311.6	2.38176	0.70752	30074.4	2.38303	0.70596	29837.2

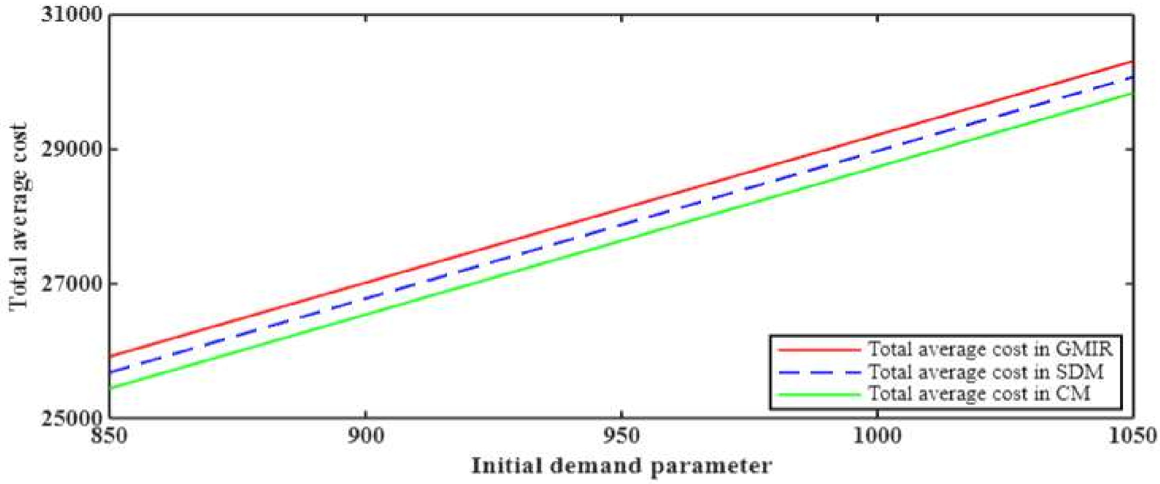

Figure 2. Variation of total avg. cost w.r.t. Initial demand parameter(α)

Table 2. Sensitivity analysis on Positive Parameter

	GIMR			SDM			CM		
β	τ	t_1	$\widetilde{TAC_g}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_S}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_C}(\tau,t_1$
(9,10,11)	2.3769	0.71285	26323.1	2.3767	0.71307	26300.5	2.3771	0.71259	26345.7
(10,11,12)	2.3771	0.71241	24966.3	2.377	0.71252	24930.8	2.3772	0.71231	25001.7
(11,12,13)	2.3773	0.71193	23609.4	2.3773	0.71188	23561.2	2.3772	0.71199	23657.6

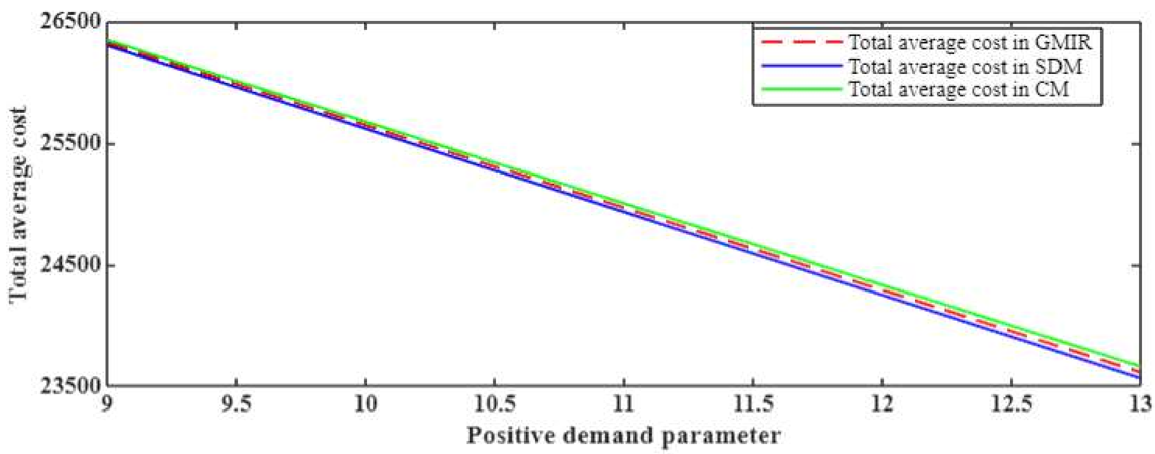

Figure 3. Variation of total avg. cost w.r.t Positive demand parameter(β)

Table 3. Sensitivity analysis on Selling Price

	GIMR			SDM			CM		
S	τ	t_1	$\widetilde{TAC_g}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_S}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_C}(\tau,t_1)$
(30,36,42)	2.3786	0.71031	23441.8	2.3793	0.7094	23285.7	2.3801	0.70848	23129.5
(36,42,48)	2.3789	0.70933	20753.7	2.38	0.70791	20572	2.3812	0.70647	20390.2
(42,48,54)	2.3793	0.70806	18.65.6	2.381	0.70598	17858.2	2.3827	0.70386	17650.9

Figure 4. Variation of total avg. cost w.r.t. Selling price(S)

Table 4. Sensitivity analysis on Shortage cost

	GIMR			SDM			CM		
C_{sc}	τ	t_1	$\widetilde{TAC_g}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_S}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_C}(\tau,t_1)$
(10,13,16)	2.3203	0.87504	33854.4	2.3207	0.87471	33698.4	2.3210	0.87437	33542.4
(13,16,19)	2.2742	1.0246	41403.6	2.2745	1.02441	41222.5	2.2748	1.02423	41041.5
(16,19,22)	2.2364	1.16281	48793.3	2.2394	1.15209	48006.2	2.2369	1.16269	48381.8

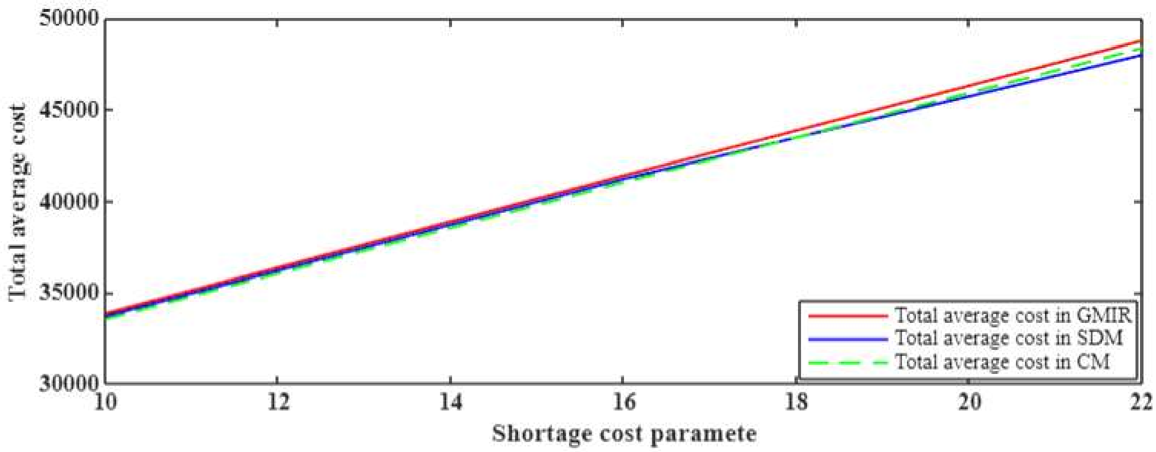

Figure 5. Variation of total avg. cost w.r.t. Shortage cost (C_{sc})

Table 5. Sensitivity analysis on Lost Sale cost

	GIMR			SDM			CM		
C_{lc}	τ	t_1	$\widetilde{TAC_g}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_S}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_C}(\tau,t_1)$
(9,12,15)	2.3813	0.70329	25782.4	2.3817	0.70278	25653	2.3821	0.70225	25523.6
(12,15,18)	2.3843	0.69546	25434.4	2.3847	0.69494	25306.2	2.3851	0.69440	25177.9
(15,18,21)	2.3874	0.68759	25086.1	2.3878	0.68706	24959	2.3881	0.68651	24831.8

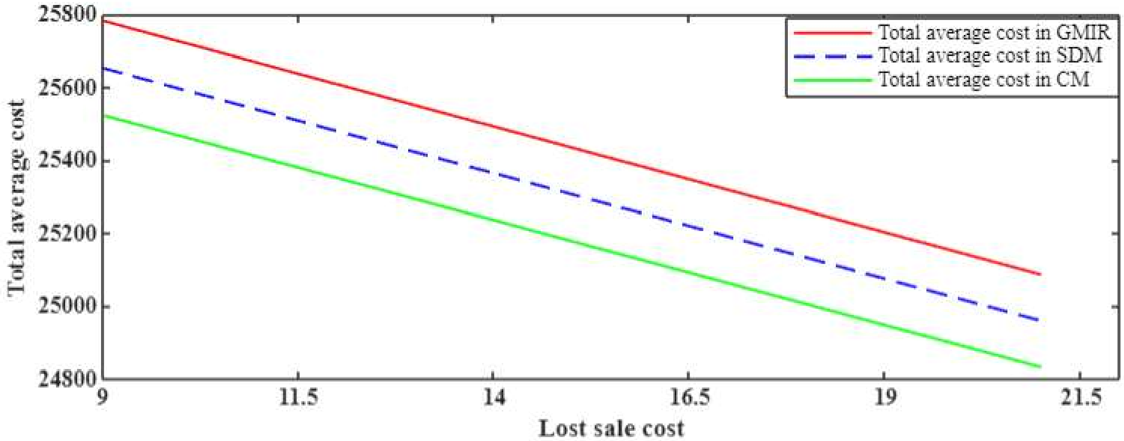

Figure 6. Variation of total avg. cost w.r.t. Lost sale cost (C_{lc})

Table 6. Sensitivity analysis on Purchasing cost

	GIMR			SDM			CM		
C_{pc}	τ	t_1	$\widetilde{TAC_g}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_S}(\tau,t_1)$	τ	t_1	$\widetilde{TAC_C}(\tau,t_1)$
(10,11,12)	2.4180	0.67237	26215.7	2.4184	0.67185	26084.7	2.4188	0.67133	25953.7
(11,12,13)	2.4529	0.63759	26292.1	2.4533	0.63706	26160.7	2.4537	0.63654	26029.4
(12,13,14)	2.4836	0.60615	26360.5	2.4840	0.60563	26228.8	2.4844	0.60510	26097.1

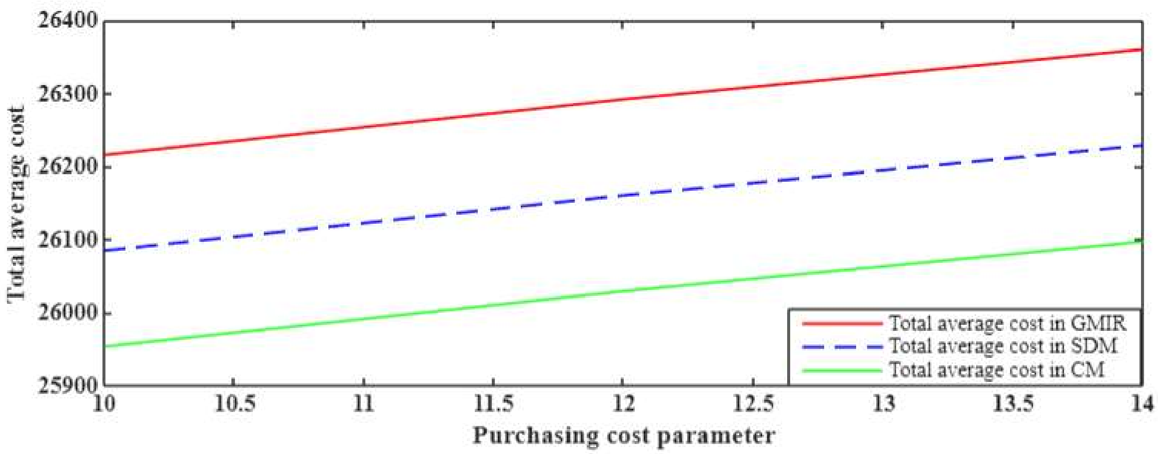

Figure 7. Variation of total avg. cost w.r.t. Purchasing cost (C_{pc})

Marginal Insights

Results are expressed in the line graphs; each line graph represents the change in inventory parameters in figures 2 to 7. Each figure consists of three-line graphs and each line graph corresponds to the total average cost which is shown by legends on the right side.

- In Table 1 and Figure 2, it is observed that while the parameter 'α' increases then 'τ', and t_1 both slightly increase in the case of GMIR, SDM but in the case of CM 'τ' decreases and 't_1' increases. Also, the total average cost $\widetilde{TAC_g}(\tau,t_1)$, $\widetilde{TAC_S}(\tau,t_1)$, and $\widetilde{TAC_C}(\tau,t_1)$ increases.
- From Table 2 and Figure 3, it is noticed that while the parameter 'β' increases then 'τ' increases in all three cases, and 't_1', decreases in all three cases. With this effect, the total average cost $\widetilde{TAC_g}(\tau,t_1)$, $\widetilde{TAC_S}(\tau,t_1)$, and $\widetilde{TAC_C}(\tau,t_1)$ decreases.
- From Table 3 and Figure 4, if the selling price 'S' increases then the preservation cost 'τ' increases in all three cases slightly and the parameter 't_1' slightly decreases in all three cases. Also, the total average cost $\widetilde{TAC_g}(\tau,t_1)$, $\widetilde{TAC_S}(\tau,t_1)$, and $\widetilde{TAC_C}(\tau,t_1)$ decreases.
- From Table 4 and Figure 5, if the shortage Cost parameter 'C_{sc}' increases then the preservation cost

'τ' decreases in all three cases, and the parameter 't_1' increases in all three cases. With this effect, the total average cost $\widetilde{TAC}_g(\tau,t_1)$, $\widetilde{TAC}_S(\tau,t_1)$, and $\widetilde{TAC}_C(\tau,t_1)$ increases.

- From Table 5 and Figure 6, It is observed that the lost sale cost parameter 'C_{lc}' increases then the preservation cost 'τ' increases in all three cases, and the parameter 't_1' decreases in all three cases. With this effect, the total average cost $\widetilde{TAC}_g(\tau,t_1)$, $\widetilde{TAC}_S(\tau,t_1)$, and $\widetilde{TAC}_C(\tau,t_1)$ decreases.
- From Table 6 and Figure 7, if the purchasing cost parameter 'C_{pc}' increases then the preservation cost 'τ' increases, but the parameter 't_1' decreases in all three cases. Also, the total average cost $\widetilde{TAC}_g(\tau,t_1)$, $\widetilde{TAC}_S(\tau,t_1)$, and $\widetilde{TAC}_C(\tau,t_1)$ decreases.

Conclusion

Mostly researchers worked in inventory modeling by assuming liner and constant holding cost with selling price and stock-based demand rate. But in real life, these quantities are not exactly linear and constant. In this novel paper, a fuzzy inventory model with a non-linear demand pattern using preservation technology with non-linear holding cost to control the deterioration rate has been developed. The proposed model is discussed crisp as well as a fuzzy environment. Since demand, selling price, holding Costs, deterioration costs, lost sale Costs, shortage costs, and purchase costs due to uncertainty, these parameters have been considered as triangular fuzzy numbers. The main objective of the study is to determine the optimum result of the fuzzy model in which the fuzzy numbers are defuzzified by graded mean Integration Representation (GMIR), Signed Distance (SDM), and Centroid methods (CM)

In this paper, we observed that due to the uncertain nature of the system parameters, the total average cost decreases in the fuzzy model as compared to the crisp model. Also, we have observed that in Centroid method gives more accurate results as compared to the GMIR and SD methods. Sensitivity analysis indicates that the total cost function is more sensitive to changes in the initial demand, shortage, and purchasing cost parameters. After analyzing the result, the decision maker can play for the optimal value for total cost and other related parameters.

The model can be used for products like fruits, and vegetables i.e. onion, potato, etc. The present model is extended by considering the demand function to be time and stock-dependent, or price-dependent under a time-dependent deterioration rate and adding some other parameters like trade credit policy, inflation, etc.

References:

1. A.K. Sahu, S. K. Indrajitsingha (2020), An optimizing inventory model for investing preservation technologies in deteriorating products with hybrid type of price and stock dependent demand rate, Int. J. Proc. Management. 15(2),.201-218,DOI:10.1504/IJPM.2020.10034304
2. Das, S.C., Zidan, A.M., Manna, A.K., Shaikh, A.A., Bhunia, A.K., (2020). An application of preservation technology in inventory control system with price dependent demand and partial backlogging, Alexandria Engineering Journal, 59(1), 1359-1365. https://doi.org/10.1016/j.aej.2020.03.006
3. Dye, C.Y., (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model, Omega, 41(5), 872-880. https://doi.org/10.1016/j.omega.2012.11.002
4. Dye, C.Y., Hsieh, T.P., (2012). An optimal replenishment policy for deteriorating items with effective investment in preservation technology, European Journal of Operational Research, 218(1), 106-112. DOI:10.1016/j.ejor.2011.10.016
5. Harris, F. W. (1915). Operations and cost, factory management. Production and Operation Management, 5(1), 1–6.
6. Hsu, P.H., Wee, H.M., Teng, H.M., (2010). Preservation technology investment for deteriorating inventory, Int. J. of Production Economics, 124(1), 388-394. https://doi.org/10.1016/j.ijpe.2009.11.034
7. Indrajitsingha, S.K., Samanta, P.N., Misra, U.K. (2018) 'A fuzzy inventory model for deteriorating items with stock dependent demand rate', Int. J. Logistics Systems and Management, ISSN(P) 1742-7967, ISSN(O) 1742-7975, Vol-30, Issue-4, pp. 538-555, DOI:10.1504/IJLSM.2018.093588
8. Indrajitsingha, S.K., Samanta, P.N., Misra, U.K., Padmini Raula (2021) 'A fuzzy model for deteriorating products using preservation technology', Indian Journal of Natural Sciences, ISSN(P) 0976- 0997, Vol-12, Issue-67, pp. 33134-33142.
9. Jani, M.Y., Betheja, M.R., Chaudhari, U., Sarkar, B., (2021). Optimal Investment in Preservation Technology for Variable Demand under Trade-Credit and Shortages. Mathematics, 9,1301. https://doi.org/10.3390/math9111301
10. A.K. Sahu, V Ganesh, S. K. Indrajitsingha(2024), Fuzzy inventory model of seasonal deteriorating products with time and price sensitive demand and holding cost under partial backlogged, Research Square Platform LLC, 2024,https://doi.org/10.21203/rs.3.rs-4187757/v1
11. Kuppulakshmi, V., Sugapriya, C., Kavikumar, J., Nagarajan, D. (2023), Fuzzy Inventory Model for Imperfect Items with Price Discount and Penalty Maintenance Cost.Mathematical Problems in Engineering, Volume 2023, Article ID 1246257, 15 pages. https://doi.org/10.1155/2023/1246257
12. Macı´as-Lo´pez, A., Ca´rdenas-Barro´n, L.E., Peimbert-Garcı´a, R.E., and Mandal, B. (2021). An Inventory Model for Perishable Items with Price-, Stock-, and Time-Dependent Demand Rate considering Shelf-Life and Nonlinear Holding Costs. Mathematical Problems in Engineering,

2021, Article ID 6630938, 36 pages. https://doi.org/10.1155/2021/6630938

13. Mishra, U., Aguilera, J.T., Tiwari, S., (2018). Retailer's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments, Mathematical Problems in Engineering, Article ID 696.2417, 1-14, https:/doi.org/10.1155/2018/696217.
14. Mishra, U., Wu, J.Z., Tseng, M-L., (2019). Effects of a hybrid-price-stock-dependent demand on the optimal solutions of a deteriorating inventory system and trade credit policy on the remanufactured product, Journal of Cleaner Production, 241(1), 1-15. DOI:10.1016/j.jclepro.2019.118282
15. Pal, H., Bardhan, S., Giri, B.C., (2018). Optimal replenishment policy for non-instantaneously permissible items with preservation technology and random deterioration start time, Int. J. of Management Science and Engineering Management, 13:3, 188-199, DOI: 10.1080/17509653.2017.1372228
16. Raula, P., Indrajitsingha, S. K., Samanta, P., Misra, U., & Raju, L. K. (2021). An EOQ Model of Selling-Price-Dependent Demand for Non-Instantaneous Deteriorating Items during the Pandemic COVID-19. *Walailak Journal of Science and Technology (WJST)*, *18*(12), Article 13398 (14 pages). https://doi.org/10.48048/wjst.2021.13398
17. Saha, S., Nielsen, I., Moon, I., (2017). Optimal retailer investments in green operations and preservation technology for deteriorating items, J. of Cleaner Production, 140(3), 1514-1527.
18. Sharma, D. K., Ompal Singh, (2024). Inventory model having preservation technology with fixed lifetime under two two level trade credit policy. Indonesian Journal of Electrical Engineering and Computer Science, Vol.35(1), pp. 610-619, ISSN: 2502-4752, DOI: 10.11591/ijeecs.v35.i1.pp610-61
19. Sindhuja. S., Arathi. P., (2023). An inventory model for deteriorating products under preservation technology with timedependent quality demand, Cogent Engineering, 10:1, 2176968, DOI: 10.1080/23311916.2023.2176968.
20. Singh, S.R., Khurana, D., Tayal, S., (2016). An economic ordered quantity model for deteriorating products with stockdependent demand with trade credit period and preservation technology, Uncertain Supply Chain Management, 4(1), 29- 42.
21. Sumeet Gill, Kumari, M. (2024), An Inventory Model to Control Deterioration Rate Using Preservation Technology with NonLinear Holding Cost,J. Electrical Systems 20-3 (2024): 5421-5433.