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Systematic Review

An Approach of Numerical Solution and Uncertainty Measure of MHD Viscous Flow Over a Shrinking Sheet with Second Order Slip Flow Model Under Fuzzy Environment

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Abstract : The specified problem for our consideration is *MHD* viscous flow over a shrinking sheet with second order slip flow model. In this paper, we investigate MHD viscous flow with a second order slip flow model over a permeable shrinking surface under fuzzy environment. The governing differential equation can be obtained by using similarity variable technique and then using the Zadeh extension theorem for the same equation to fuzzify. The $\alpha - cut$ technique is used to show the validation for the uncertainty of the equation of the motion. The effect of magnetic parameter M, mass transfer parameter s, first order velocity slip parameter γ and second order velocity slip parameter δ has been investigated in fuzzy environment. It is observed that the effect of uncertainty due to change of the parameter M and s not for the parameter σ and δ. A suitable range in which magnetic parameter, mass transfer parameter, first order velocity slip parameter and second order velocity slip parameter is evaluated that gives the accuracy in the velocity profile and finally the solution is carried out and presented graphically. It has been found that in crisp case whenever M decreases the velocity also decreases. A Comparison of skin friction coefficient is also shown for crisp and fuzzy valued.

Keyword : Triangular Fuzzy Set, Fuzzified, Crisp, Magnetic Parameter

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Introduction :

The motion of rigid bodies or fluids, vehicle, washing machine etc. are uncertain due to uncertainty of equations of motion, the variables and the parameters involved in the mathematical model. Hence there are enough scope to fuzzify the variables, parameters and the boundary conditions of the flow model of the fluid flow due to a moving boundary. In this chapter we have considered a non-linear differential equation of motion with one parameter in the equations and three parameters in the boundary conditions. The specified problem for the discussion is MHD viscous flow over a shrinking sheet with second order slip flow model. Sakiadis (1990) introduce the concept on boundary layer on a stretching surface with a constant speed. But Wang (1992) discovered the exact solution of Navier Stokes equation of the Sakiadis proposed problem with lot of assumption. Subsequently many researchers have done lots of work on the said problem with constrain to make the equation of motion are exact. This assumption leads to some fuzzy to the originally proposed problems. Such problems are models to differential equation having some prescribe boundary conditions. The solution and behaviors of such differential equation are fuzzy in nature. So it is important to fuzzify the differential equations appeared in the governing equation of motion and to solve the system under fuzzy environment. Chang and Zadeh (1965) invent the theory of fuzzy valued function, following Dubois and Prade (1990-1998), Puri and Ralescu (1983), Kaleva (1998), Seikkala (1998) and many more have done some work related to it. On the field of fuzzy differential equation research are running the studies now a days in full Monty (2000 to cont.) Magnetohydrodynamic (MHD) flow is traditionally different from general flow. In recent years lots of research are going on this area. In this paper we shall study MHD viscous flow over a shrinking sheet with second order slip flow. The equations governing the motion are fuzzified by using Zadeh extension principle and then numerical solution is carried out by developing computer codes for the problem. The crisp solution and mid value solution of the triangular fuzzified system of equations is in good agreement. The solution and effect of slip parameter, magnetic parameter and suction parameter are investigated under fuzzy environment with details through graph.

2. FORMULATION OF THE PROBLEM

We consider a two dimensional laminar flow of magnetized viscous fluid over a continuously shrinking sheet. The shrinking velocity of the sheet is $U_w = -U_0 x$, where U_0 is a constant and $v_w = v_w(x)$ is wall mass transfer velocity. The x −axis is along the shrinking surface in the direction opposite the sheet motion and y −axis is perpendicular to it. By these assumption the corresponding Navier-Stoke's equations for the present problem can be summarized by the following set of equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad \ldots\ldots\ldots (1)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \sigma\frac{B^2 u}{\rho} \quad \ldots\ldots\ldots (2)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \ldots\ldots\ldots (3)$$

The boundary conditions of the above problems are given by

$$u(x,0) = U_0(x) + U_{slip}$$

$$v(x,0) = v_w$$

$$u(x,0) = 0 \quad \ldots\ldots... (4)$$

Where u and v are the components of the velocity in the x and y directions respectively, ϑ is the kinematic viscosity,p is the fluid pressure, ρ is the fluid density and $U_{slip}$is second order velocity slip which is valid for any arbitrary Knudsen number, K_n and is given by

$$U_{slip} = \frac{2}{3}(\frac{3-\alpha l^3}{\alpha} + \frac{3}{2}\left(\frac{1-l^2}{K_n}\right)\lambda\frac{\partial u}{\partial y} - \frac{1}{4}(l^4 + \frac{2}{{K_n}^2}(1-l^2))\lambda^2\frac{\partial^2 u}{\partial y^2}$$

$$= A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2}. \quad ..\ldots\ldots (5)$$

Where $l = \min\left[\frac{1}{K_n} - 1\right]$, $0 \le \alpha \le 1$ is the momentum accommodation coefficient and $\lambda > 0$ is the molecular mean free path.according to the definition of l it is observed that $0 \le l \le 1$, for any value of K_n. The stream function and similarity variable would be of the following form

$$\psi(x,y) = f(\eta)x\sqrt{vU_0}, \eta(x,y) = y\sqrt{\frac{U_0}{v}} \quad \ldots\ldots\ldots (6)$$

With this definition the components of velocities are

$$u = f'(\eta)xU_0, v = -f(\eta)\sqrt{vU_0} \quad \ldots\ldots\ldots (7)$$

The wall mass transfer velocity becomes

$$v_w = f(0)\sqrt{vU_0}, \quad \ldots\ldots\ldots (8)$$

Using Equation (3.7) and (3.8) in (3.3), we get

$$\frac{p}{\rho} = v\frac{\partial v}{\partial y} - \frac{v^2}{2} + constant \quad \ldots\ldots\ldots (9)$$

Also by using equations (7)-(9) in equation (2), we have

$$f''' + ff'' - {f'}^2 - M^2 f' = 0 \quad \ldots\ldots\ldots (10)$$

Where $M^2 = \frac{\sigma B^2}{\rho\mu_0}$. And the boundary conditions are

$$f'(0) = -1 + \gamma f''(0) + \delta f'''(0) \quad \ldots\ldots\ldots (11)$$

$$f(0) = s \quad \ldots\ldots\ldots (12)$$

$$f(1) = 0 \quad \ldots\ldots\ldots (13)$$

Where s is the mass transfer parameter, γ is the first order velocity slip parameter, $\gamma = A\sqrt{\frac{U_0}{v}}$ and $\delta = \frac{BU_0}{v} < 0$ is the second order velocity parameter. The pressure term can be obtained from equation (9).

3. CONVERSION OF THE BASIC EQUATIONS INTO FUZZIFIED FORM

All the equations (1)-(13), all the in crisp form, now we have use the Zadeh extension principle to convert the equation (10) into fuzzy differential equation we have,

$$\widehat{f'''} + \hat{f}\widehat{f''} - \widehat{f'^2} - \widehat{M^2}\widehat{f'} = \hat{0} \qquad \dots\dots\dots (14)$$

And the boundary conditions in Fuzzified form are as follows

$$\widehat{f'(0)} = \widehat{-1} + \hat{\gamma}\widehat{f''(0)} + \hat{\delta}\widehat{f'''(0)} \qquad \dots\dots\dots (15)$$

$$\widehat{f(0)} = \hat{s} \qquad \dots\dots\dots (16)$$

$$\widehat{f(1)} = \hat{0} \qquad \dots\dots\dots (17)$$

Considering the fuzzy number are triangular, then we have the equation (14) is as follows

$$[\underline{f'''}, f''', \overline{f'''}] + [\underline{f}, f, \overline{f}]\,[\underline{f''}, f'', \overline{f''}] - \left[\underline{f'^2}, f'^2, \overline{f'^2}\right] - [\underline{M^2}, M^2, \overline{M^2}]\,[\underline{f'}, f', \overline{f'}] = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\underline{f'''}, f''', \overline{f'''}] + [\min T, T_0, \max T] - \left[\underline{f'^2}, f'^2, \overline{f'^2}\right] - [\min K, K_0, \max K] = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow \left[\underline{f'''} + \min T, f''' + T_0, \overline{f'''} + \max T\right] - \left[\underline{f'^2} + \min K, f'^2 + K_0, \overline{f'^2} + \max K\right] = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\underline{f'''} - \overline{f'^2} + \min T - \max K, f''' - f'^2 + \min T - \max K, \underline{f'^2} - \overline{f'''} + \min K - \max T]$$

$$= [\underline{0}, 0, \overline{0}] \qquad \dots\dots\dots (18)$$

where $T = \overline{f}\overline{f''}, \overline{f}\underline{f''}, \underline{f}\overline{f''}, \underline{f}\underline{f''}$, and $T_0 = ff''$

$$K = \overline{M^2}\overline{f'}, \overline{M^2}\underline{f'}, \underline{M^2}\overline{f'}, \underline{M^2}\underline{f'}, \text{ and } K_0 = M^2 f'' \qquad \dots\dots\dots (19)$$

The above equation can also be splitting like

$$\underline{f'''} - \overline{f'^2} + \min T - \max K = \underline{0} \qquad \dots\dots\dots (20)$$

$$f''' - f'^2 + \min T - \max K = 0 \qquad \dots\dots\dots (21)$$

$$\underline{f'^2} - \overline{f'''} + \min K - \max T = \overline{0} \qquad \dots\dots\dots (22)$$

With the boundary condition of the equations (15-17) can be in the form of

$$\overline{f'(0)} = \overline{-1} + \max W + \max U \qquad \dots\dots\dots (23)$$

$$f'(0) = -1 + W_0 + U_0 \qquad \dots\dots\dots (24)$$

$$\underline{f'(0)} = \underline{-1} + \min W + \min U \qquad \dots\dots\dots (25)$$

$$\overline{f(0)} = \overline{s} \qquad \dots\dots\dots (26)$$

$$f(0) = s \qquad \dots\dots\dots (27)$$

$$\underline{f(0)} = \underline{s} \qquad \dots\dots\dots (28)$$

$$\overline{f(1)} = \overline{0} \qquad \dots\dots\dots (29)$$

$$f(1) = 0 \qquad \dots\dots\dots (30)$$

$$\underline{f(1)} = \underline{0} \qquad \dots\dots\dots (31)$$

where

$$U = \overline{\gamma}\overline{f''(0)}, \overline{\gamma}\underline{f''(0)}, \underline{\gamma}\overline{f''(0)}, \underline{\gamma}\underline{f''(0)}$$

$$W = \overline{\delta}\overline{f'''(0)}, \overline{\delta}\underline{f'''(0)}, \underline{\delta}\underline{f'''(0)}, \underline{\delta}\overline{f'''(0)}$$

$$U_0 = \gamma f''(0) \text{ and } W_0 = \delta f'''(0)$$

4. DEFINITION OF SKIN FRICTION (C_f)

The physical quantities of principal interest is the skin friction coefficient C_f, which are defined as

$$Re_x^{1/2} C_f = f''(0)$$

where

$Re_x = \frac{u_x(x)x}{v}$ is the local Reynolds number.

5. RESULT AND DISCUSSIONS

The Eq.s (20-22) together with boundary condition (23-31) is solved numerically by using finite difference scheme. The discretized Fuzzified equation are solved using an iterative method based on Gauss Seidel method by developing suitable codes in python. Result is obtained for different value for the parameters M=1, s=1, $\gamma = 0$, $\delta = -.04$and for different $\alpha - cut$ of the Fuzzified equations (23-31).

(In each of the following graph pink curve, yellow curve and grey curve represent the right value, mid value and left value solution)

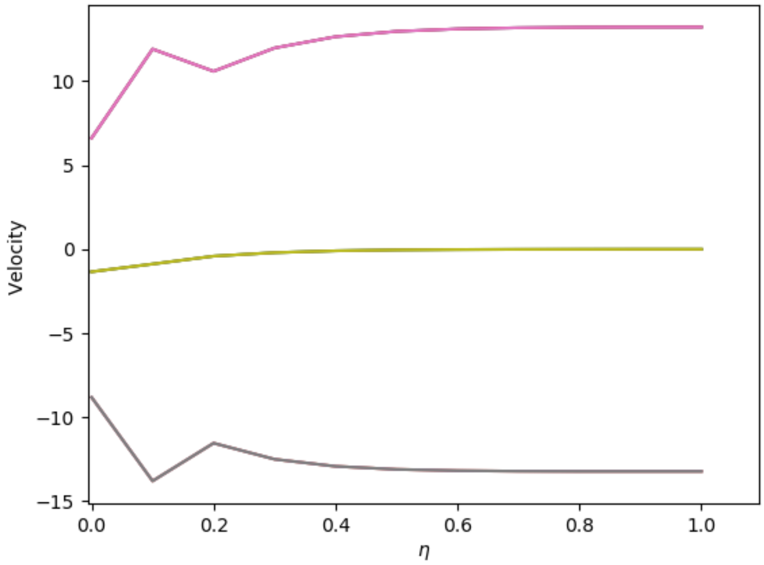

Fig 1: Fuzzified Velocity profile for M=1, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .2$

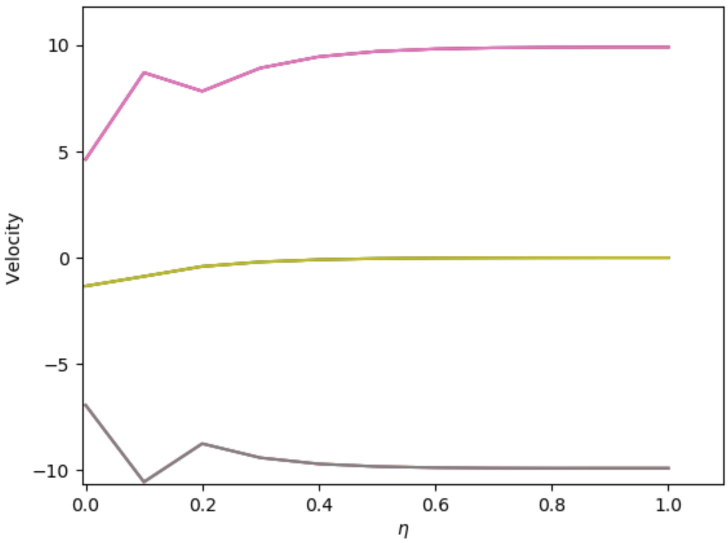

Fig 2: Fuzzified Velocity profile for M=1, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .4$

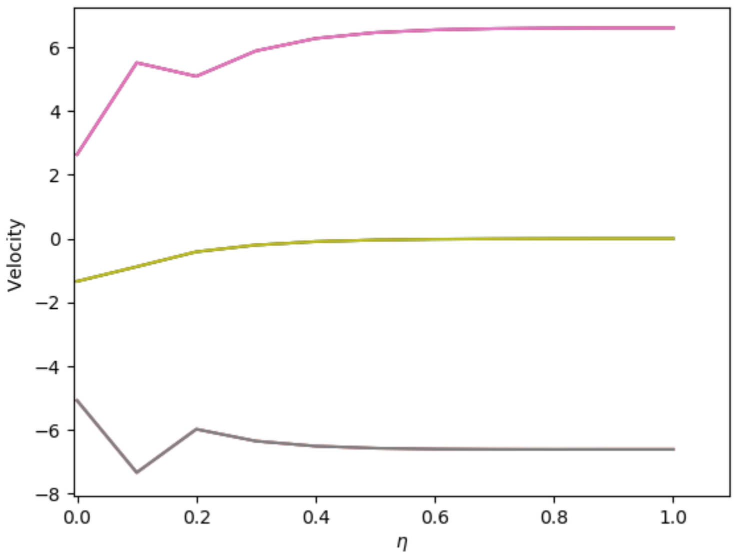

Fig 3: Fuzzified Velocity profile for M=1, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .6$

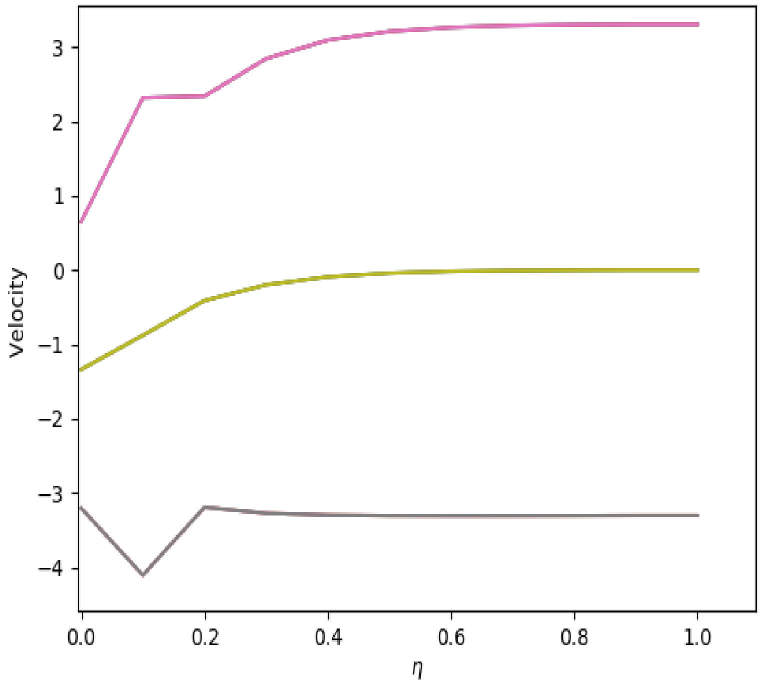

Fig 4: Fuzzified Velocity profile for M=1, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .8$

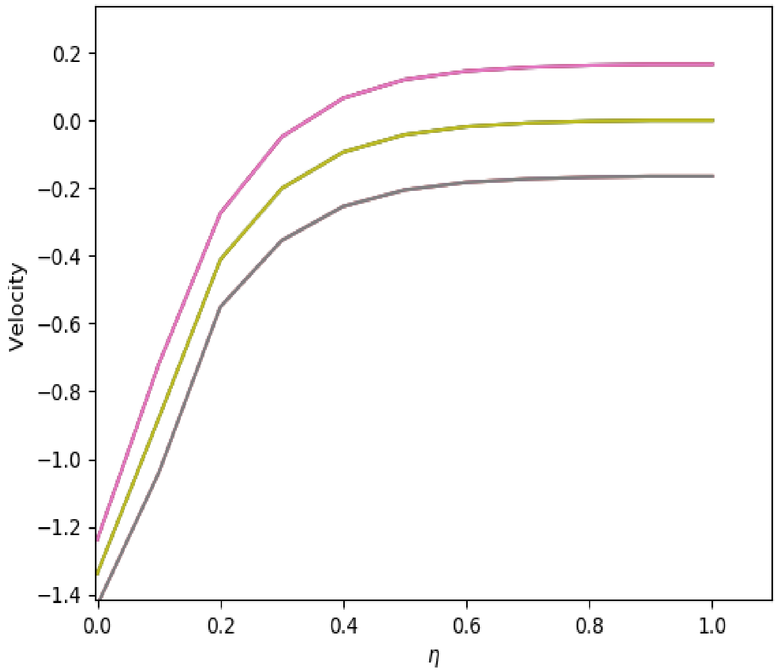

Fig 5: Fuzzified Velocity profile for M=1, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .9$

The Figs. (1-5) are the fuzzified velocity profile for the value of the parameter M=1, s=1, $\gamma = 0$, $\delta = -0.04$ and for different $\alpha - cut$. It is observed from above figures that the fuzzified velocity profile shows deflection on the curves. When we are going to the close values of the $\alpha - cut = 1$ (Crisp solution) the deflection on the curves are decreases and the flow pattern is same in both the right values and as well as left values of the fuzzified velocity profile. Which is the clear indicate of the validation of uncertainty in the above problem.

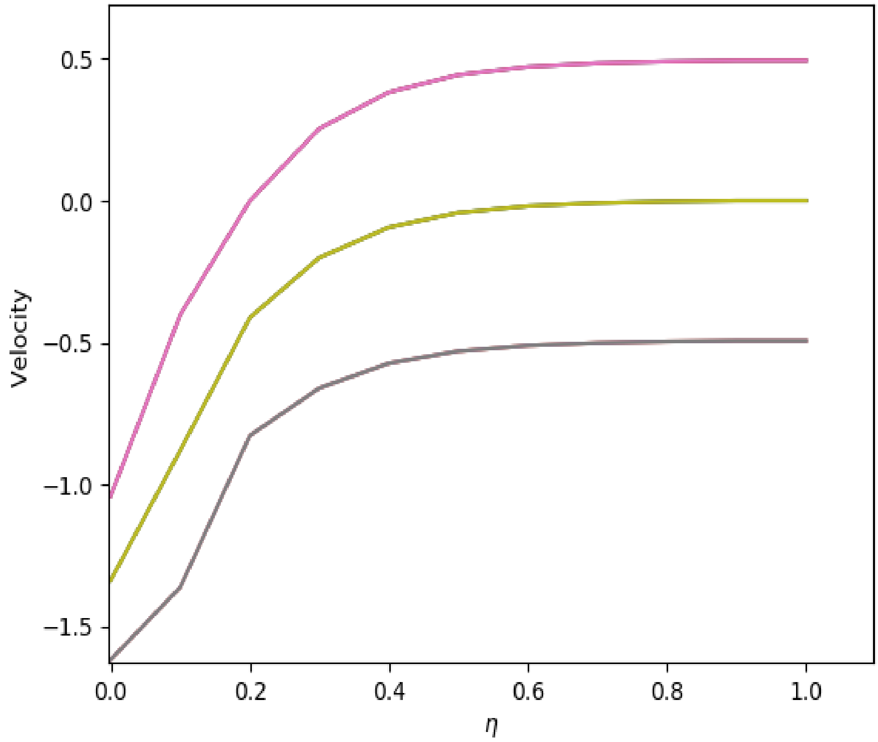

Fig 6: Fuzzified velocity profile for **M= .01,.02,.03**, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .9$

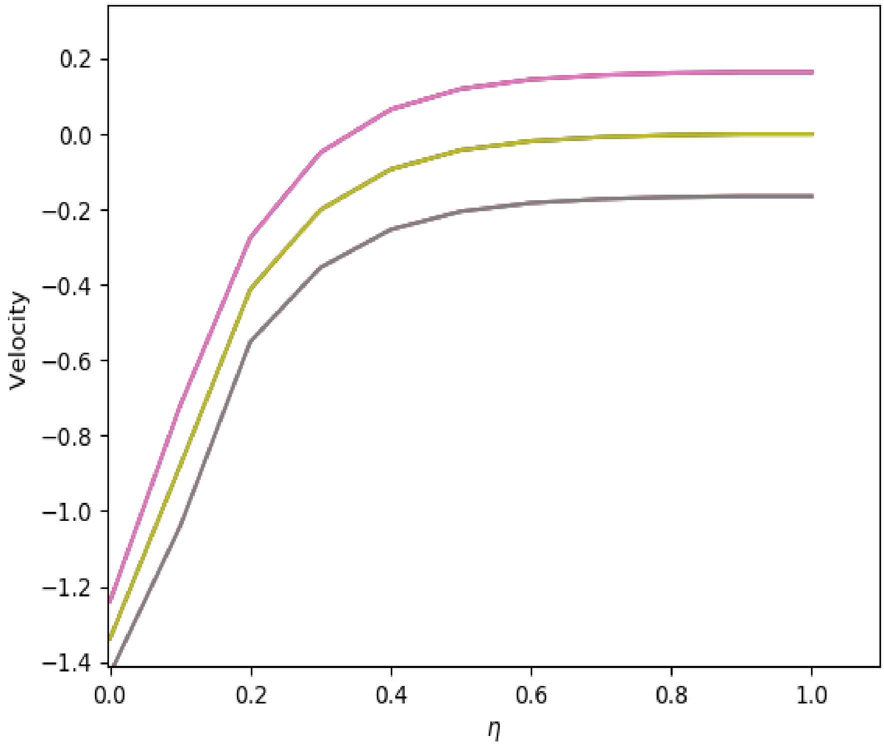

Fig 7: Fuzzified Velocity profile for **M= 0.1,0.2,0.3**, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .9$

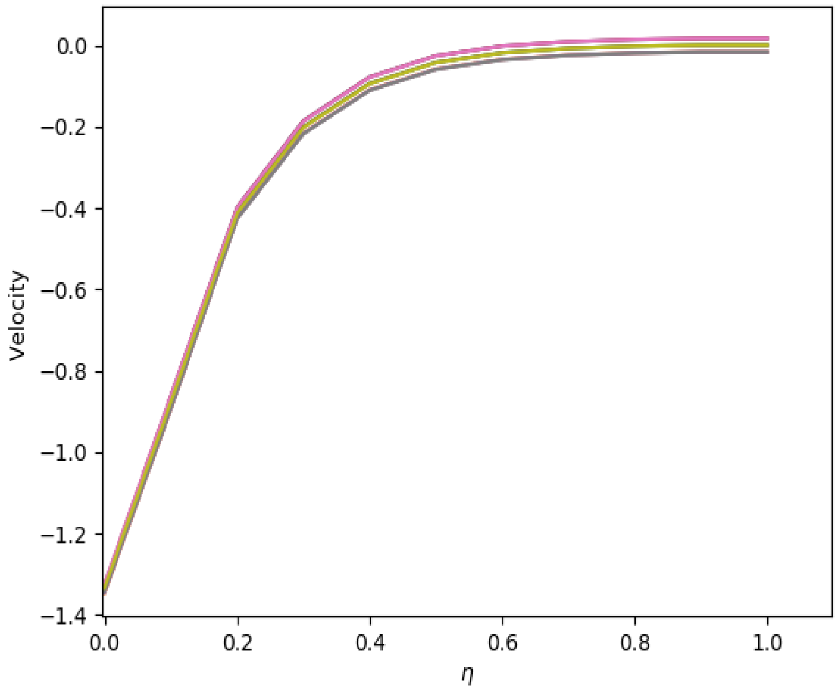

Fig 8: Fuzzified Velocity profile for **M= 1,2,3**, s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - \ cut = .9$

Here, Fig (3.6-3.8) are the fuzzified velocity profiles for the different values of the parameter M and specific value of the parameter s=1, $\gamma = 0$, $\delta = -.04$ and $\alpha - cut = .9$. It is found that the fuzzified velocity profile shows more deflection when the value of$M$ is smaller whereas higher value of M give us less deflection of the curves i.e. more accuracy in the result or we may say less chance of uncertainty in the solution of the velocity profile.

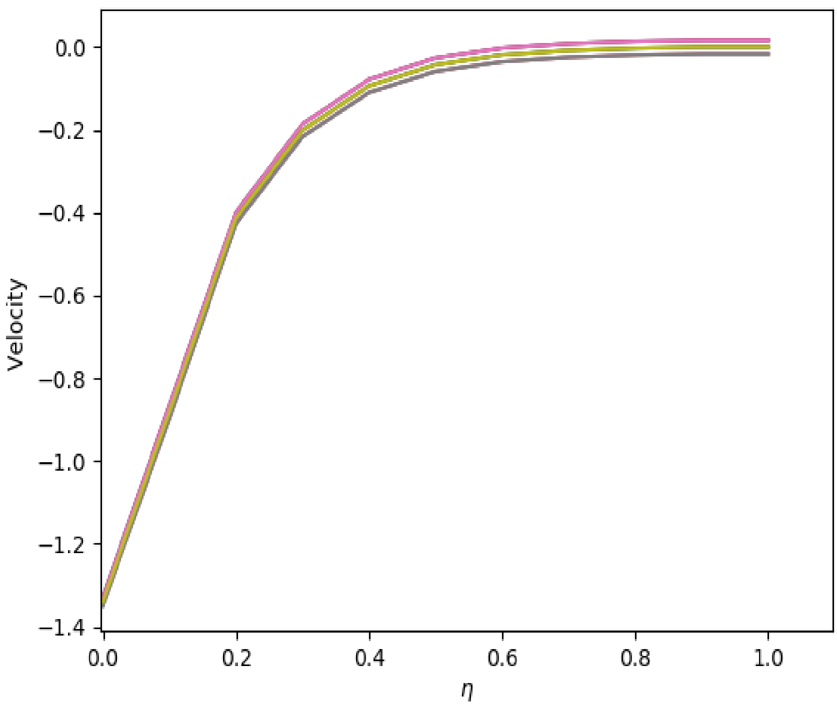

Fig 9: Fuzzified velocity profile for M= 1, **s= 0.001, .002,.003**, $\gamma = 0.1$, $\delta = -.04$ and $\alpha - cut = .9$

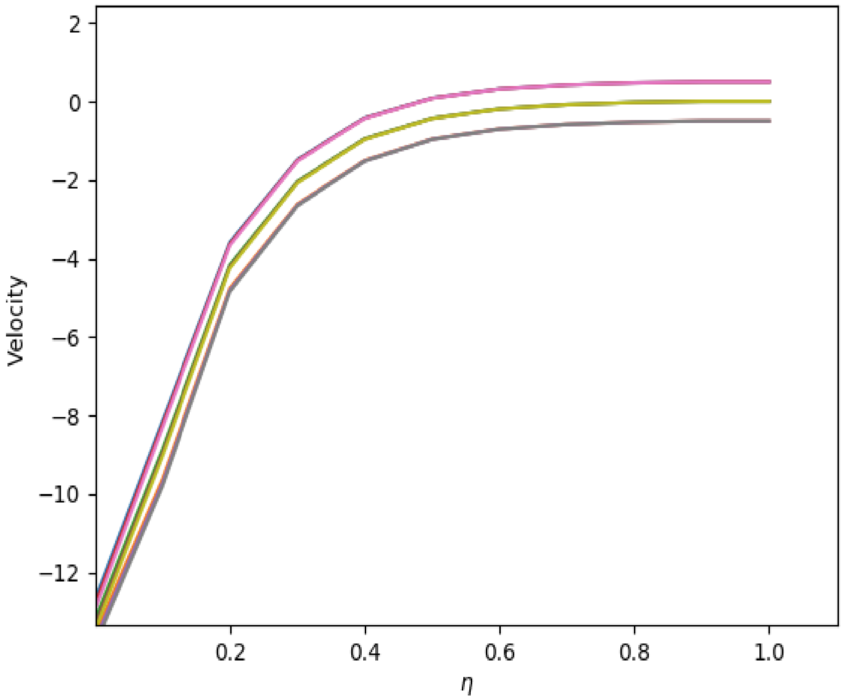

Fig 10: Fuzzified velocity profile for M= 1, **s= 0.01, .02,.03**, $\gamma = 0.1$, $\delta = -.04$ and $\alpha - cut = .9$

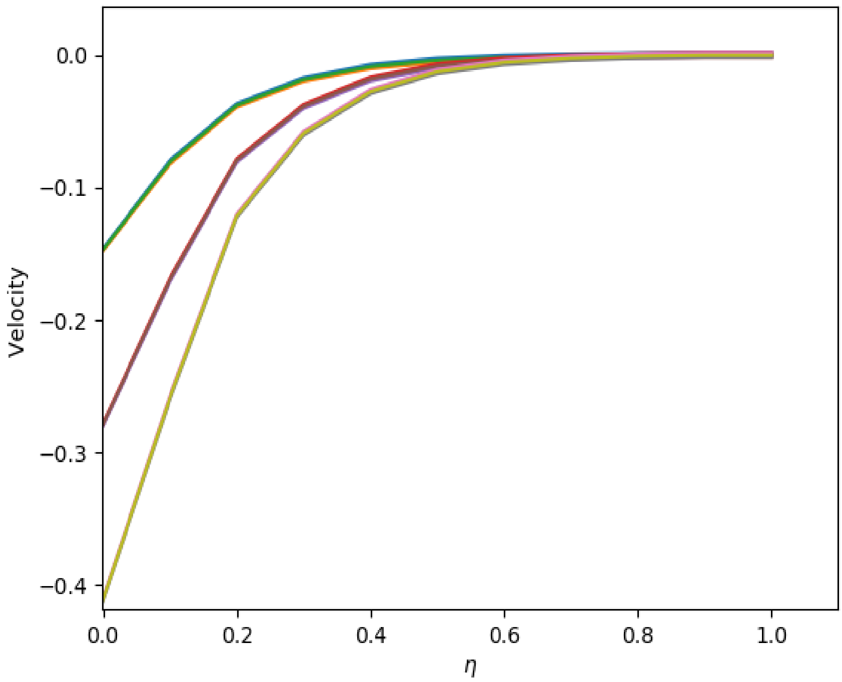

Fig 11: Fuzzified velocity profile for M= 1, **s= 0.1, .2,.3**, $\gamma = 0.1$, $\delta = -.04$ and $\alpha - cut = .9$

The Figs. (9-11) are the fuzzified velocity profile for the values of the parameters M= 1, $\gamma = 0.1$,$\delta = -.04$ and $\alpha - cut = .9$and different values of s. It is found that the smaller values of the parameter s shown less deflection on the fuzzified velocity profile whereas higher values of the parameter s shown more deflection. Which indicates that for the smaller values of s, the solution is more accurate than the higher value of s.

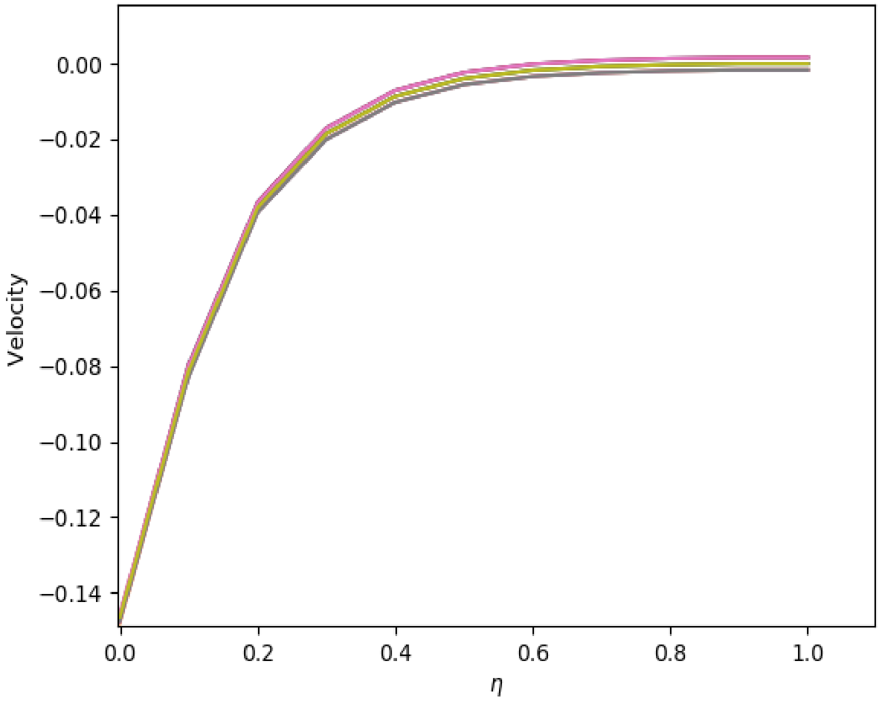

Fig 12: Fuzzified velocity profile for M= 1, s= 0.001,$\boldsymbol{\gamma = 0.1, .2, 3}$, $\delta = -.04$ and $\alpha - cut = .9$

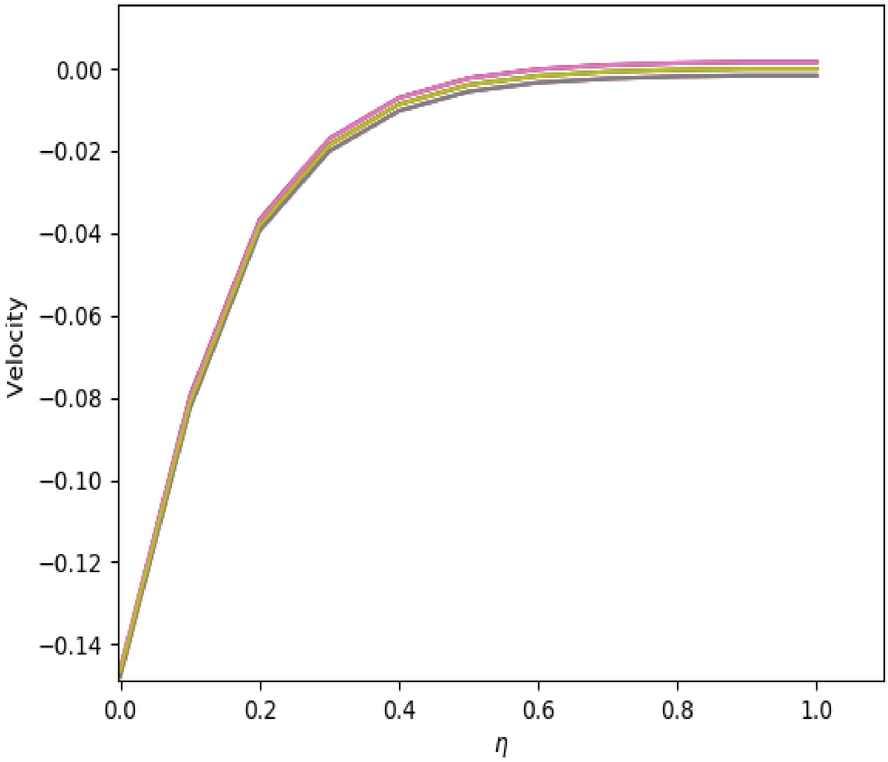

Fig 13: Fuzzified velocity profile for M= 1, s= 0.001,$\boldsymbol{\gamma = 1, 2, 3}$, $\delta = -.04$ and $\alpha - cut = .9$

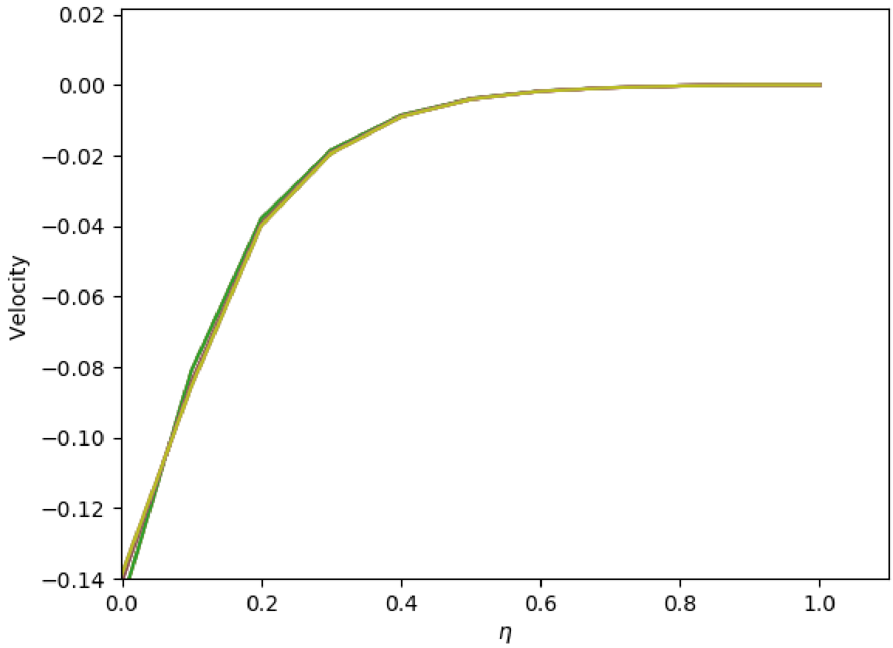

Fig 14: Fuzzified velocity profile for M= 1, s= 0.001,$\gamma = 0.1$, $\boldsymbol{\delta = -.4, -.65, -0.9}$ and $\alpha - cut = .9$

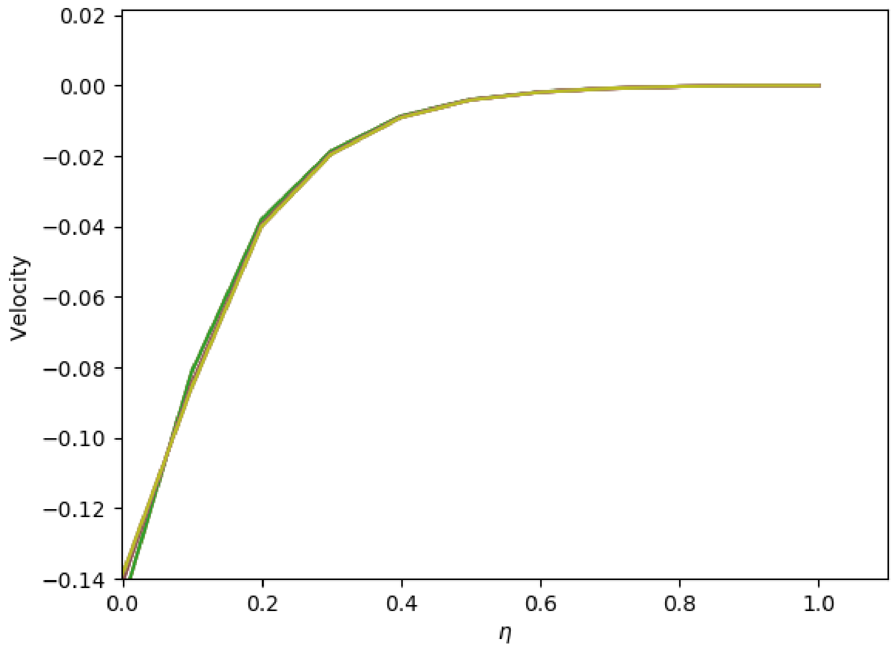

Fig 15: Fuzzified Velocity profile for M= 1, s= 0.001,$\gamma = 0.1$, $\boldsymbol{\delta = -.04, -.08, -0.12}$ and $\alpha - cut = .9$

Also, the Figs. (11-15) are the fuzzified velocity profiles for the values of the parameters given by $\alpha - cut = .9$ M= 1, s= 0.001,and different values of γ and δ It is reflected from the above figure that there is no significant deflection occurs in the Fuzzified velocity profiles for the change of the parameter γ and δ. Which conclude that the effect of the uncertainty doesn't due the change of the parameter γ and δ. Now, we choose the suitable values of different M and s for the crisp temperature profile are shown graphically below

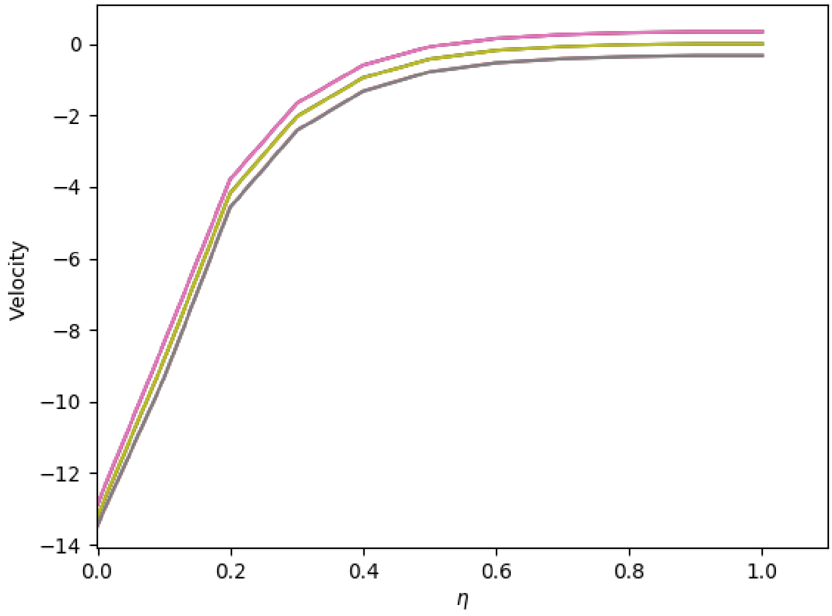

Fig 16: Crisp velocity profile for **M= 2,4,6**, s=0.001, $\gamma = 0$, $\delta = -.04$

The Fig. 16 represent the crisp velocity profile for different values of $M = 2{,}4\ and\ 6$ by the curve of color pink, yellow and violet respectively with the value of the parameter s=0.001, $\gamma = 0$, $\delta = -0.04$. It is observed from the curve that the high magnetic effect implies increase of the velocity.

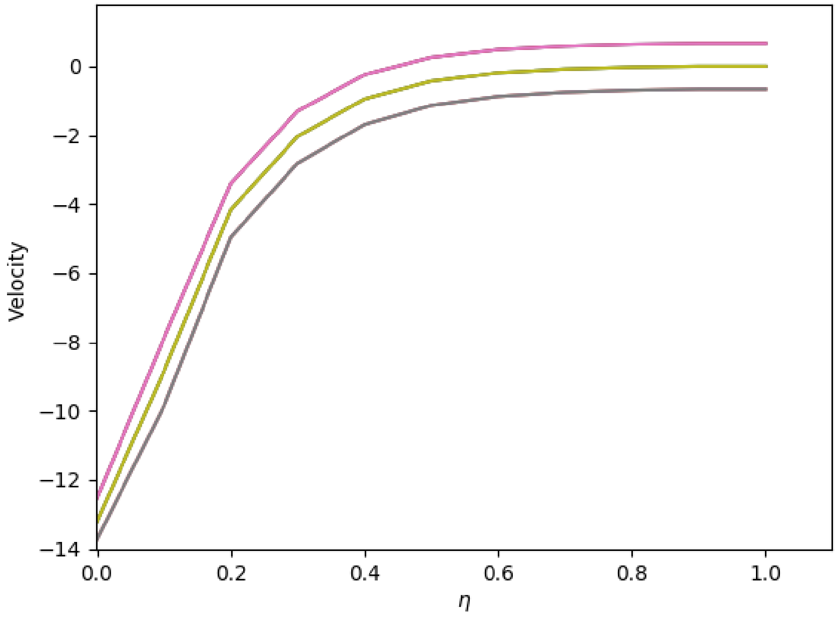

Fig 17 : Crisp velocity profile for M= 1, **s= 0.002, .004,.006**, $\gamma = 0.1$, $\delta = -.04$

The Fig. 17 represent the crisp velocity profile for different values of $s = 0.002, 0.004\ and 0.00\ 6$ by the curve of color pink, yellow and violet respectively with the value of the parameter M=1, $\gamma = 0$, $\delta = -.04$. It is observed from the curve that the behavior of the velocity profile depicted with varying s.

6. COMPARISON OF SKIN FRICTION COEFFICIENT C_f

One of the important parameter in fluid flow problem are the skin friction coefficient C_f define in the section 3. We have computed these parameters for different values of the parameter M with fix Reynold number which are given in the following table.

Comparison of fuzzy and crisp values of skin friction coefficient C_f

M	C_f			
	Crisp	**Fuzzify**		
0	0.925042	1.1367686	0.925042	0.7133154
.1	0.920932	1.1255507	0.920932	0.7163133
.2	0.914067	1.109677	0.914067	0.718457
.3	0.8879903	1.0742536	0.8879903	0.701727

From the above table we observed that the effect of parameter M on the skin friction coefficient C_f in crisp sense are decrease with increasing value of M.The crisp values and the mid values of the fuzzify values are same which indicates the validity of the result. . The effect of fuzzification are also observed from the above table.

7. CONCLUSION

In this paper, we studied about the magneto hydrodynamic viscous flow over a Shrinking sheet with second order slip flow model under fuzzy environment. The effect of the magnetic parameter M, Suction parameter$s$, first order velocity slip parameter γ and second order velocity slip parameter δ on the uncertainty of the solution profile have been considered under fuzzy environment. The numerical result have been obtained by developing computer codes on PYTHON. Thus, we conclude the followings from the above discussion:

- The involvement of uncertainty in the equation of motion of this problem.
- Change of the Magnetic parameter is one of the causes of the uncertainty in the solution of the velocity profile and the higher value of the Magnetic parameter gives more accuracy in the solution of the problems.
- Change of the Suction parameter is one of the causes of the uncertainty in the solution of the velocity profile and the smaller value of the suction parameter gives more accuracy in the solution of the problems.
- The first order velocity slip parameter γ and second order velocity slip parameter δ doesn't effect the uncertainty.
- In crisp case whenever M decreases velocity is also decreases.
- The effect of the fuzzification is observed from the values of the skin friction co-efficient.

Data availability

No data was used for the research described in the article.

Declarations Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this paper. Our manuscript has no associated data.

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Ethical Conduct.

Ethical approval was not required.

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